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The Banzhaf Value When Some Players Are Incompatible

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Abstract Situations in which there are incompatible players were first modeled by TU games in Bergantiños et al. (1993). In this paper a modification of the Shapley value for such situations is defined. In Alonso-Meijide and Casas-Méndez (2007) a modification of the Public Good Index for simple games with incompatibilities was introduced. In this paper, we define a modification of the Banzhaf value for this kind of situations and provide two characterizations of it. We illustrate this value with a real-world example taken from the political field.

Keywords cooperative game, Banzhaf value, incompatibilities, axiomatic characterization.

1. Introduction

An n-person cooperative game with transferable utility models conflict situations where the agents involved can achieve binding agreements and the

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joint utility can be split among the players. One of the most interesting question that appears is which amount will be allotted to each player.

The most important answer was provided by Shapley (1953) and is known as the Shapley value. Another interesting value was introduced by Banzhaf (1965), initially proposed in the context of voting games and later on extended to general TU games by Owen (1975). A player's assessment by means of these values is the average of his marginal contribution to any coalition to which the player does not belong to. But they differ in the weights associated to each marginal contribution. The non-normalized Banzhaf value (from now on we will call it the Banzhaf value) arises when we assume that every player is equally likely to enter any coalition. The Shapley value considers that every player is equally likely to join any coalition of a given size and that all such coalitions are equally likely. The main difference between these two values is that the Shapley value is efficient, while the Banzhaf value satisfies the total power property. Thus, the Shapley value distributes the total utility among players while the total amount that players get from Banzhaf's allocation depends on the structure of the TU game.

In order to represent cooperative situations more accurately, more sophisticated models appear. Several modifications of the two previous values for such models have been proposed in the literature. Some of them take into account the possibility that some players may be more likely to act together than others as it usually happens in the case of the deputies of a Parliament who are members of the same political party (Owen (1977; 1982), and Alonso-Meijide and Fiestras-Janeiro (2002)). In other cases, the communication among players is restricted by the existence of a communication graph representing that communication, and hence cooperation, only occurs among agents who are connected by a path in the graph (Myerson (1977) and Owen (1986)). In Myerson (1977), a modification of the Shapley value (the Myerson value) is defined and characterized using the properties of efficiency in components and fairness. In Owen (1986), a modification of the Banzhaf value is defined for situations where the communication is restricted by a graph. This value was characterized in Alonso-Meijide and Fiestras-Janeiro (2006) using the properties of component total power and fairness.

In this paper, we consider situations where some players are incompatible, that is, there are players that cannot cooperate at all. The incompatibilities among players are described by means of a graph. A link between two agents indicates that they are incompatible. These situations are studied in Carreras (1991), Bergantiños (1993), and Carreras and Owen (1996). In the last paper, a modification of the Shapley value for these situations is defined and characterized. The characterization is based on an i-efficiency property which establishes that incorporating a link to the incompatibility graph hurts or benefits the agents involved in the same way. In Vázquez-Brage (1998) situations with a priori unions and incompatible players are studied. In Alonso-Meijide and Casas-Méndez (2007) the Public Good Index is defined and characterized when some players are incompatible for the family of simple games.

The main objective of our paper is to define and characterize a modification of the Banzhaf value when some players are incompatible. In Section 2, we give the main notions related with TU games and games with incompatible players. In Section 3, we define the modification of the Banzhaf value for games with incompatible players. Finally, we analyze the Parliament of the Basque Country.

2. Preliminaries

2.1 *TU games*

A game with transferable utility (or TU game) is a pair (N,v), where $N = \{1,\ldots,n\}$ is the set of players and v, the characteristic function, is a real valued function on $2^N = \{S|S \subseteq N\}$ with $v(\emptyset) = 0$. A subset $S \subseteq N$ is called a coalition. We denote by $G(N)$ the set of TU games with player set N. We will use shorthand notation and write $S \cup i$ for the set $S \cup \{i\}$ and $S \backslash i$ for the set $S \backslash \{i\}$.

A *null player* in a game $(N,v) \in G(N)$ is a player $i \in N$ such that $v(S \cup i) = v(S)$ for all $S \subseteq N \backslash i$. Two players $i, j \in N$ are *symmetric* in a game $(N,v) \in G(N)$ if $v(S \cup i) = v(S \cup j)$ for all $S \subseteq N \backslash \{i,j\}$.

Given a family of games $H \subseteq G(N)$, a *value* on H is a function f, which assigns to every game $(N,v) \in H$ a vector $(f_1(N,v),\ldots,f_n(N,v)) \in \mathbb{R}^n$, where the real number $f_i(N,v)$ is the payoff of the player i in the game (N,v) according to f. It is useful to single out a list of standard desirable properties a value may satisfy.

A value f satisfies the *transfer* property if

$$f(N,v) + f(N,w) = f(N,v \vee w) + f(N,v \wedge w)$$

for every $(N,v),(N,w) \in H$ such that $(N,v \vee w),(N,v \wedge w) \in H$, where $(v \vee w)(S) = \max\{v(S),w(S)\}$ and $(v \wedge w)(S) = \min\{v(S), w(S)\}$ for every $S \subseteq N$.

A value f satisfies the *null player* property if $f_i(N,v) = 0$ for every $(N,v) \in H$ and every null player $i \in N$.

A value f is *anonymous* if $f_{\sigma(i)}(N,v) = f_i(N,\sigma v)$ for every $(N,v) \in H$ and every $i \in N$, where σ is a permutation on N such that $(N,\sigma v) \in H$ and $(N,\sigma v)$ is defined by $\sigma v(S) = v(\sigma(S))$ for every $S \subseteq N$.

A value f is *efficient* if $\sum_{i\in N} f_i(N,v) = v(N)$ for every $(N,v) \in H$.

A value f satisfies the *total power* property if
$\sum\limits_{i\in N} f_i(N,v) = \frac{1}{2^{n-1}} \sum\limits_{i\in N} \sum\limits_{S\subseteq N\setminus i} [v(S\cup i) - v(S)]$ for every $(N,v) \in H$.

The total power property states that the total payoff obtained by players is the sum of all marginal contributions of every player normalized by 2^{n-1}. It is clear that if a value on $G(N)$ is efficient then this value cannot satisfy the total power property.

The most well-known values on $G(N)$ are the Shapley value (Shapley (1953)) and the Banzhaf value (Banzhaf (1965) and Owen (1975)). There is a vast literature concerning characterizations of both the Shapley and the Banzhaf values. In Feltkamp (1995) characterizations of both values are provided in terms of some of the previous properties. We recall them here:

The unique value f on $G(N)$ that satisfies *efficiency, anonymity, the null player property, and the transfer property* is the Shapley value. Given a game $(N,v) \in G(N)$, this value assigns to each player $i \in N$ the real number:

$$\varphi_i(N,v) = \sum_{S\subseteq N\setminus i} \frac{s!\,(n-s-1)!}{n!} \left(v(S\cup i) - v(S)\right).$$

The unique value f on $G(N)$ that satisfies *the total power property, anonymity, the null player property, and the transfer property* is the Banzhaf value. Given a game $(N,v) \in G(N)$, this value assigns to each player $i \in N$ the real number:

$$\beta_i(N,v) = \sum_{S\subseteq N\setminus i} \frac{1}{2^{n-1}} \left(v(S\cup i) - v(S)\right).$$

Notice that the Shapley value satisfies efficiency while the Banzhaf value satisfies the total power property.

2.2 *TU games with incompatibilities*

Let $N = \{1,2,\ldots,n\}$ be a finite set. An undirected graph without loops on N is a set of unordered pairs of different elements. These pairs $(i:j)$ are called links. Note that $(i:j) = (j:i)$. We denote by I^N the complete incompatibility graph on N and by $GR(N)$ the set of all undirected graphs on N, that is:

$$I^N = \{(i:j) \mid i \in N, j \in N, i \neq j\} \text{ and } GR(N) = \{I | I \subseteq I^N\}.$$

Let us take $I \in GR(N)$. Given a pair of agents $i, j \in N$, we say that i and j are incompatible if $(i:j) \in I$.

A *TU game with incompatibilities* is a triple (N, v, I) where $(N, v) \in G(N)$ and $I \in GR(N)$. The set of all such games with common player set N will be denoted by $I(N)$.

Given $(N, v, I) \in I(N)$, we will say that a coalition $S \subseteq N$ is *I-admissible* if there are not incompatible players contained on it. The case $I = \emptyset$, represents the situation in which all coalitions are I-admissible.

If $I \in GR(N)$, for any coalition $S \subseteq N$, we denote by $P(S, I)$ the set of all partitions of S whose classes are I-admissible coalitions. Given $I \in GR(N)$, I^c denotes the dual graph of I, i.e., $I^c = \{(i:j) \mid i, j \in N,\ i \neq j,\ (i:j) \notin I\}$. For any $S \subseteq N$ and any $I \in GR(N)$, we say that $i, j \in S$ are connected in S by I if I induces a path contained on S connecting i and j, and we denote by S/I the set of connected components of S (maximal subsets of connected elements of S) by the graph I. Note that S/I is a partition of S.

For any $i \in N$, we denote by I^{*i} the element of $GR(N)$ obtained from I when player i becomes incompatible with any other player $j \in N \setminus i$, i.e.,

$$I^{*i} = I \cup \{(i:j) \in I^c \mid j \in N\}.$$

Given a game with incompatibilities (N, v, I), Bergantiños (1993) defined the associated I-restricted game.

Definition 2.1 Given a TU game with incompatibilities $(N, v, I) \in I(N)$, we denote by $(N, v^I) \in G(N)$ the *I-restricted game* whose characteristic function is given by,

$$v^I(S) = \max_{P \in P(S,I)} \sum_{T \in P} v(T)\ , \text{ for all } S \subseteq N.$$

A value on $I(N)$ is a function f, which assigns to every game $(N, v, I) \in I(N)$ a vector $(f_1(N, v, I), \ldots, f_n(N, v, I)) \in \mathbb{R}^n$, where the real number $f_i(N, v, I)$ is the payoff of the player i in the game (N, v, I) according to f.

In Bergantiños et al. (1993), a value on $I(N)$ is introduced. The proposed value is closely related to the Shapley value. Moreover, the value is characterized with the following two properties.

A value on $I(N)$ satisfies *i-component efficiency* if

$$\sum_{i\in S} f_i(N,v,I) = \max_{P\in P(S,I)} \sum_{T\in P} v(T),$$

for every $(N,v,I) \in I(N)$ and every $S \in N/I^c$.

This property guarantees that each maximal admissible component gets the largest available amount since it is formed by compatible agents who can cooperate.

A value on $I(N)$ satisfies *i-fairness* if

$$f_i(N,v,I) - f_i(N,v,I \cup (i:j)) = f_j(N,v,g) - f_j(N,v,I \cup (i:j)),$$

for every $(N,v,I) \in I(N)$ and every $(i:j) \in I^c$.

This property establishes that incorporating a link to the incompatibility graph hurts or benefits the agents involved in the same way.

The proposed value is characterized in the next result.

Theorem 2.2 Bergantiños et al. (1993). There is a unique value on $I(N)$ satisfying i-component efficiency and i-fairness. This value is given by:

$\rho(N,v,I) = \varphi(N,v^I)$ for every $(N,v,I) \in I(N)$.

In Alonso-Meijide and Casas-Méndez (2007) a new characterization of this value is obtained. This characterization is based on the axiom of i-balanced contributions.

A value on $I(N)$ satisfies *i-balanced contributions* if

$$f_i(N,v,I) - f_i(N,v,I^{*j}) = f_j(N,v,I) - f_j(N,v,I^{*i}),$$

for every $(N,v,I) \in I(N)$ and every $i,j \in N$.

This property says that the loss or gain of player i when player j becomes incompatible with any other player is equal to the loss or gain of player j when player i becomes incompatible with any other player.

Theorem 2.3 Alonso-Meijide and Casas-Méndez (2007). The value ρ is the unique value on $I(N)$ satisfying i-component efficiency and i-balanced contributions.

3. The Banzhaf value for games with incompatible players

In this section we propose an extension of the Banzhaf value as a new value on $I(N)$. We characterize the proposed value, and the characterizations reveal that the properties satisfied by the new value and the value ρ are very similar. The difference lies on the fact that the former satisfies the i-total power property and the latter the i-component efficiency property. This is in correspondence with the previously mentioned distinction between the Banzhaf value and the Shapley value previously mentioned.

Using the arguments of the total power and i-component efficiency properties jointly, we propose the i-total power property.

A value defined on $I(N)$ satisfies *i-total power* if

$$\sum_{i\in S} f_i(N,v,I) = \frac{1}{2^{s-1}} \sum_{i\in S} \sum_{L\subseteq S\setminus i} \left[v^I(L\cup i) - v^I(L)\right],$$

for every $(N,v,I)\in I(N)$ and every $S\in N/I^c$.

The Banzhaf value for games with incompatibilities β^I is the natural extension of the Banzhaf value to the class $I(N)$ using the associated I-restricted game, i.e.,

$$\beta^I(N,v,I) = \beta(N,v^I).$$

The next result shows that the Banzhaf value satisfies the properties introduced above.

Lemma 3.1 The Banzhaf value for games with incompatibilities satisfies i-fairness, i-balanced contributions, and i-total power properties.

Proof
i-fairness. Let $i,j\in N$ such that $(i:j)\notin I$, then

$$\begin{aligned} &2^{n-1}\left[\beta_i^I(N,v,I) - \beta_i^I(N,v,I\cup(i:j))\right] \\ =& \sum_{S\subseteq N\setminus\{i,j\}} \left[v^I(S\cup i\cup j) - v^I(S\cup j) + v^I(S\cup i) - v^I(S)\right] \\ -& \sum_{S\subseteq N\setminus\{i,j\}} \left[v^{I\cup(i:j)}(S\cup i\cup j) - v^{I\cup(i:j)}(S\cup j) + v^{I\cup(i:j)}(S\cup i) - v^{I\cup(i:j)}(S)\right]. \end{aligned}$$

Since for all $S\subseteq N\setminus\{i,j\}$,

$$v^I(S) = v^{I\cup(i:j)}(S),\ v^I(S\cup i) = v^{I\cup(i:j)}(S\cup i), \text{ and } v^I(S\cup j) = v^{I\cup(i:j)}(S\cup j),$$

then

$$\begin{aligned}&\beta_i^I(N,v,I)-\beta_i^I(N,v,I\cup(i:j))\\=&\frac{1}{2^{n-1}}\sum_{S\subseteq N\setminus\{i,j\}}\left[v^I(S\cup i\cup j)-v^{I\cup(i:j)}(S\cup i\cup j)\right]\\=&\beta_j^I(N,v,I)-\beta_j^I(N,v,I\cup(i:j)).\end{aligned}$$

i-balanced contributions. Let $i,j\in N$, then

$$\begin{aligned}&2^{n-1}\left[\beta_i^I(N,v,I)-\beta_i^I(N,v,I^{*j})\right]\\=&\sum_{S\subseteq N\setminus\{i,j\}}\left[v^I(S\cup i\cup j)-v^I(S\cup j)+v^I(S\cup i)-v^I(S)\right]\\-&\sum_{S\subseteq N\setminus\{i,j\}}\left[v^{I^{*j}}(S\cup i\cup j)-v^{I^{*j}}(S\cup j)+v^{I^{*j}}(S\cup i)-v^{I^{*j}}(S)\right].\end{aligned}$$

Since for all $S\subseteq N\setminus j$,

$$v^I(S)=v^{I^{*j}}(S)\text{ and }v^{I^{*j}}(S\cup j)=v^I(S)+v(j),$$

then

$$\begin{aligned}&2^{n-1}\left[\beta_i^I(N,v,I)-\beta_i^I(N,v,I^{*j})\right]\\=&\sum_{S\subseteq N\setminus\{i,j\}}\left[v^I(S\cup i\cup j)-v^I(S\cup i)-v^I(S\cup j)+v^I(S)\right]\\=&2^{n-1}\left[\beta_j^I(N,v,I)-\beta_j^I(N,v,I^{*i})\right].\end{aligned}$$

i-total power. Let $S\in N/I^c$ and take $(N,v^{I,S})\in G(N)$ defined as follows,

$$v^{I,S}(T)=\max_{P\in P(T\cap S,I)}\sum_{L\in P}v(L)\text{ for all }T\subseteq N.$$

First of all we will see that $v^I=\sum_{S\in N/I^c}v^{I,S}$.

Let $P\in P(T,I)$ and $L\in P$. As L is I-admissible, it follows that L is a connected component of N by the graph I^c. Then,

$$\sum_{L\in P}v(L)=\sum_{L\in P}\sum_{S\in N/I^c}v(L\cap S)=\sum_{S\in N/I^c}\sum_{L\in P}v(L\cap S)\leq\sum_{S\in N/I^c}v^{I,S}(T),$$

since $\{L\cap S|\ L\in P\}\in P(T\cap S,I)$. Then,

$$v^I(T)\leq\sum_{S\in N/I^c}v^{I,S}(T),\text{ for every }T\subset N.$$

On the other hand, let $T \subset N$. We build a partition $P \in P(T,I)$ as follows. $P = \cup_{S \in N/I^c} P_S$ with $P_S \in P(T \cap S, I)$ such that,

$$v^{I,S}(T) = \sum_{L \in P_S} v(L) = \max_{Q \in P(T \cap S, I)} \sum_{L \in Q} v(L), \text{ for every } S \in N/I^c.$$

Then,

$$v^I(T) \geq \sum_{L \in P} v(L) = \sum_{S \in N/I^c} v^{I,S}(T).$$

Hence, the stated equality follows.

Given $S \in N/I^c$ and using the additivity property of the Banzhaf value, β, we have

$$\begin{aligned}
&\sum_{j \in S} \beta_j^I(N, v, I) = \sum_{j \in S} \beta_j(N, v^I) \\
=&\sum_{j \in S} \sum_{T \in N/I^c} \beta_j(N, v^{I,T}) = \sum_{T \in N/I^c} \sum_{j \in S} \beta_j(N, v^{I,T}).
\end{aligned}$$

For all $T \in N/I^c$ and all $i \in N \setminus T$, i is a null player in $(N, v^{I,T})$. Since β satisfies the null player property,

$$\sum_{j \in S} \beta_j^I(N, v, I) = \sum_{T \in N/I^c} \sum_{j \in S} \beta_j(N, v^{I,T}) = \sum_{j \in S} \beta_j(N, v^{I,S}) = \sum_{j \in S} \beta_j(S, v^{I,T}).$$

Finally, taking into account that β satisfies the total power property,

$$\begin{aligned}
&\sum_{j \in S} \beta_j^I(N, v, I) = \sum_{j \in S} \beta_j(N, v^I) \\
=&\sum_{j \in S} \beta_j(S, v^{I,T}) = \frac{1}{2^{s-1}} \sum_{j \in S} \sum_{L \subseteq S \setminus j} [v^I(L \cup j) - v^I(L)].
\end{aligned}$$

□

Next, we provide a characterization of the Banzhaf value for games with incompatibilities.

Theorem 3.2 The Banzhaf value for games with incompatibilities, β^I, is the unique value on $I(N)$ satisfying i-total power and i-balanced contributions.

Proof
Existence It was proved in Lemma 3.1.

Uniqueness Let f be an allocation rule satisfying the properties and $(N,v,I^N) \in I(N)$. Then, $N/(I^N)^c = \{\{1\},\{2\},\ldots,\{n\}\}$. Since f satisfies i-total power we have,

$$f(N,v,I^N) = (v(1),\ldots,v(n)),$$

and hence f is unique. Suppose that there are two different values f^1 and f^2 satisfying the properties. Then, there is $(N,v,I) \in I(N)$ such that $f^1(N,v,I) \neq f^2(N,v,I)$ and $I \neq I^N$. Hence, we can take $I \in GR(N)$ with the maximum number of links for which the inequality holds. Let $i \in N$ such that $f_i^1(N,v,I) \neq f_i^2(N,v,I)$.

If for all $j \in N \setminus i$, $(i:j) \in I$, then $\{i\} \in N/I^c$ and applying the i-total power property we come to contradiction.

If there is $j \in N \setminus i$ such that $(i:j) \notin I$. Then by i-balanced contributions and the maximality of I,

$$\begin{aligned} &f_i^1(N,v,I) - f_j^1(N,v,I) = f_i^1(N,v,I^{*j}) - f_j^1(N,v,I^{*i}) \\ =&f_i^2(N,v,I^{*j}) - f_j^2(N,v,I^{*i}) = f_i^2(N,v,I) - f_j^2(N,v,I) \end{aligned} \tag{1}$$

Moreover, let $S \in N/I^c$ such that $i \in S$, then $(i:j) \notin I$ for all $j \in S$ and there are $\{i_1,i_2,\ldots,i_k\} \subseteq S$ such that $i = i_1$, $j = i_k$, and $(i_l : i_{l+1}) \notin I$ for all $l = 1,\ldots,k-1$. Hence, by (1) we have,

$$\begin{array}{rcl} f_{i_1}^1(N,v,I) - f_{i_2}^1(N,v,I) & = & f_{i_1}^2(N,v,I) - f_{i_2}^2(N,v,I) \\ & \vdots & \\ f_{i_{k-1}}^1(N,v,I) - f_{i_k}^1(N,v,I) & = & f_{i_{k-1}}^2(N,v,I) - f_{i_k}^2(N,v,I). \end{array}$$

Adding up both sides,

$$f_i^1(N,v,I) - f_j^1(N,v,I) = f_i^2(N,v,I) - f_j^2(N,v,I). \tag{2}$$

On the other hand, using the i-total power property we have,

$$\sum_{i \in S} f_i^1(N,v,I) = \frac{1}{2^{s-1}} \sum_{i \in S} \sum_{T \subseteq S \setminus i} \left[v^I(T \cup i) - v^I(T) \right] = \sum_{i \in S} f_i^2(N,v,I). \tag{3}$$

By (2) and (3) it follows,

$$s f_i^1(N,v,I) = s f_i^2(N,v,I),$$

hence, we come to contradiction. □

In a similar way, we can obtain a characterization of the value by means of the i-fairness property instead of the i-balanced contributions property. This result is presented in the next theorem without proof.

Theorem 3.3 The Banzhaf value for games with incompatibilities, β^I, is the unique value on $I(N)$ satisfying i-total power and i-fairness.

4. Application to the Basque Country Parliament

The Parliament of the Basque Country, one of Spain's seventeen regions, is constituted by 75 members. Since most decisions are taken by majority, the characteristic function of the game played by the parties with parliamentary representation is as follows, unity for any coalition adding up 38 or more members, and zero for the rest. We will consider the situation in the Parliament after the elections in November 1986, because it has been studied before in Carreras and Owen (1996) using the Shapley value and its modification ρ and in Alonso-Meijide and Casas-Méndez (2007) using the Public Good Index and its modification taking into account an incompatibility graph. The Parliament was composed by 19 members of the Spanish socialist party PSE, 17 members of the Basque nationalist conservative party PNV, 13 members of the Basque nationalist social democrat party EA, 13 members of the Basque nationalist left-wing party HB, 9 members of the Basque nationalist moderated left-wing party EE, 2 members of the Spanish conservative party CP, and 2 members of the Spanish centrist party CDS. In the papers mentioned above, taking into account the behavior of the parties and the declarations made by the representatives of the parties involved, an incompatibility graph compound by the following links is given:

$$(PSE:HB),(PSE:CP),(HB:EE),(HB:CP),(HB:CDS).$$

For a more detailed description of each party and their political positions the reader is referenced to Carreras and Owen (1996).

In the original simple game there are twelve possible minimal winning coalitions, but when we consider the I-restricted game with the incompatibility graph only six minimal winning coalitions are feasible.

In Table 1, we present the Banzhaf value, β, and the Banzhaf value for games with incompatibilities, β^I.

The depicted results are similar to those presented in Carreras and Owen (1996) and Alonso-Meijide and Casas-Méndez (2007). PNV ranks first, even though PSE has more seats. PNV and EA are the only parties which

Party	Seats	β	β^I
PSE	19	.4688	.40625
PNV	17	.4688	.53125
EA	13	.2812	.40625
HB	13	.2812	.09375
EE	9	.2812	.28125
CP	2	.0312	0
CDS	2	.0312	.09375

Table 1 — The Banzhaf and Incompatibility Banzhaf values

increase their power significantly. CP becomes a null player because his rejection to PSE and HB.

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